Continuum mechanics (ME/AE/BME 536)

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> There is nothing that can be said by mathematical symbols and relations which cannot also be said by words. The converse, however, is false. Much that can be and is said by words cannot successfully be put into equations, because it is nonsense.
>
> — C. A. Truesdell

Course description

Cartesian tensors, transformation laws, basic continuum mechanics concepts; stress, strain, deformation, constitutive equations. Conservation laws for mass, momentum, energy. Applications in solid and fluid mechanics.s

Course Objective

This course provides the fundamental concepts and methods used in the mathematical modeling of mechanical systems (i.e., solids and fluids). Many material systems, regardless of their molecular structure, can be considered *continuous* from a macroscopic viewpoint. This simple abstraction is the cornerstone of most of the mathematical models of materials used in engineering.

The course will cover the fundamental aspects of continuum theories:

(i) kinematics (geometrical description of deformation);

(ii) basic balance laws; and

(iii) constitutive theories for both fluids and solids.

Kinematics is presented in its full nonlinear form. Small strain kinematics is derived by linearization. Balance laws will be derived according to both the Lagrangian and Eulerian frameworks. Applications will include both nonlinear and linear elasticity as well as fluid mechanics. In the case of elastic solids, the analysis of material symmetry will be covered so as to be applicable to anisotropic materials.

One important objective of this course is for the students to become proficient in reading and understanding the common notations used in the continuum mechanics literature. Therefore, both index and direct (coordinate-free) notations are described and used. This objective goes along with that of developing enough mathematical proficiency to check the accuracy of the typical derivations found in the literature.

Recommended References

- (GUR) Morton E. Gurtin, Eliot Fried, and Lallit Anand, *The Mechanics and Thermodynamics of Continua*, Cambridge University Press, 2010.
- (SPE) A. J. M. Spencer, *Continuum Mechanics*, Dover Publishing, 2004.
- (MAL) L. E. Malvern, *Introduction to the Mechanics of a Continuous Medium*, Englewood Cliffs (NJ), Prentice-Hall, 1969.
- (CHA) P. Chadwick, *Continuum Mechanics: Concise Theory and Problems*, Dover Publishing, 1999 (first edition: Wiley, 1976).
- (WU) H.C. Wu, *Continuum Mechanics and Plasticity*, Chapman and Hall/CRC, 2004 (Solids, Plasticity).
- (DIM) Y. I. Dimitrienko, *Nonlinear Continuum Mechanics and large Inelastic Deformations*, Springer, 2011 (Solids).
- (CHA) E.W.V. Chaves, *Notes on Continuum Mechanics*, Springer, 2013 (Solids, Plasticity, Damage mechanics).
- (LAI) W.M. Lai, D. Rubin, Erhard Krempl, *Introduction to Continuum Mechanics*, Elsevier, 4th edition, 2009 (Fluids).
- (BOW) R. M. Bowen, *Introduction to Continuum Mechanics for Engineers*, Plenum Press, 1989. `http://www1.mengr.tamu.edu/rbowen/` (Thermodynamics).
- (TAD) E.B. Tadmor, R.E. Miller, R.S. Elliot, *Continuum Mechanics and Thermodynamics*, Cambridge University Press, 2012 (Thermodynamics).
- (TRU) C. Truesdell and W. Noll, *The Non-Linear Field Theories of Mechanics*, Springer, 3rd edition, 2004 (Mathematics).
- (TAL) Y.R. Talpaert, *Tensor Analysis and Continuum Mechanics*, Springer, 2003 (Mathematics).
- (ROM) G. Romano, R Barretta, Continuum Mechanics on manifolds, 2009 (Mathematics, Exterior Calculus).

The topics decorated by ("topic") are discussed in more detailed in references therein. Also, some useful course notes for students' reference are,

- (ABEa & ABEb) R. Abeyaratne, Massachusetts Institute of Technology, *Lecture Notes on The Mechanics of Elastic Solids: Vol I A Brief Review of Some Mathematical Preliminaries & Vol II Continuum Mechanics.*
- (SAO) V.E.Saouma, University of Colorado Boulder, *Continuum Mechanics and Elements of Elasticity Structural Mechanics.*

Grading Scheme and Exams

The overall grade will be based on

1. homework, worth (55% +5%) Extra 5% is for challenge problems.
2. a semester project, worth 20%; and
3. the final exam, worth 25%.

NOTE: It is a requirement that the same mathematical notation used in class be used in all homework assignments and exams.

The semester project will require each student to write a paper that deals with topics covered in class. The intent is to encourage synthesis of materials available in the literature into the framework developed in class. The scope can be either literature-review or research based; however, each student must submit a single page project proposal to the instructor (by October 16, at the latest), outlining the technical merit of the project. The due date for the project will be the final day of class. Time permitting, each student may be asked to give a 15 minute presentation of the project to the class.

Course Outline

1. **Mathematical Preliminaries**
   (a) Direct and indicial notations
   (b) Vector spaces
   (c) Tensor algebra
      i. Components of a basis, contravariant and covariant components
      ii. Coordinate transformation
      iii. Tensor operations (sum, product, contraction, *etc.*)
   (d) Calculus of vector and tensor fields
      i. Tensor differentiations (Grad, Div, Curl)
      ii. Tensor integration and divergence theorems
      iii. Curvilinear coordinate systems
2. **Kinematics**
   (a) Body, deformation, and Lagrangian and Eulerian configurations
   (b) Transformation of length, area, volume, and orientation
   (c) Finite and infinitesimal strain tensors
   (d) Motion: Objective rates of change
3. **Kinetics**
   (a) Force and traction, stress tensor
   (b) Principal stresses / tensor invariants
   (c) Stress transformation / Mohr's circle
4. **Balance laws and thermodynamics**
   (a) Conserved quantity, flux density and source term
   (b) Transport theorem and localization theorem
   (c) Balance laws in Lagrangian and Eulerian configurations
   (d) Balance of mass, linear and angular momentum, and energy
   (e) Thermodynamics energy potentials
   (f) First law of thermodynamics (balance of energy)
   (g) Second law of thermodynamics (entropy production inequality)
   (h) Balance laws for arbitrary domains in spacetime
   (i) Singular Surfaces and Jump Conditions
5. **Constitutive Equations**

(a) Derivation of constitutive equations from thermodynamic energy potentials

(b) Frame-invariance (objectivity)

(c) Material symmetry groups and representation theorems

(d) Specific constitutive theories

  i. Hyperelasticity and linearized solid mechanics

  ii. Isotropic material, examples from anisotropic materials

  iii. Brief reference to rate effects, plasticity, and damage mechanics

  iv. Fluids constitution equations

Topics in "brief" will only be briefly discussed.